These exercises were originally intended for high school students in a summer session, but enough theory is included to make them appropriate for engineering students in physics or a first course in dynamics. High school students without exposure to calculus can perform the simulations and learn about friction, sliding, and rigid body rotations, while more advanced students can perform the analyses to verify the simulation results. Please feel free to copy or modify these exercises to fit your needs, and let me know if you have comments, corrections, or suggestions. If you would like an electronic copy to modify, please let me know.

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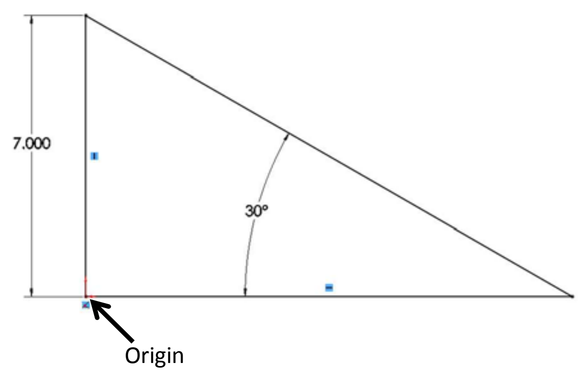
In these exercises, we will examine problems involving objects sliding and/or rolling down inclined planes.

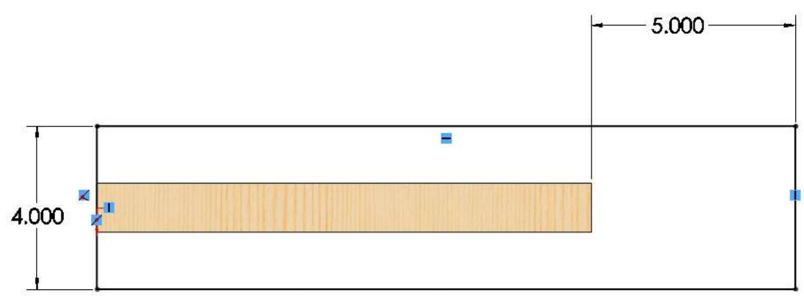
<u>Creating the SolidWorks Models</u>

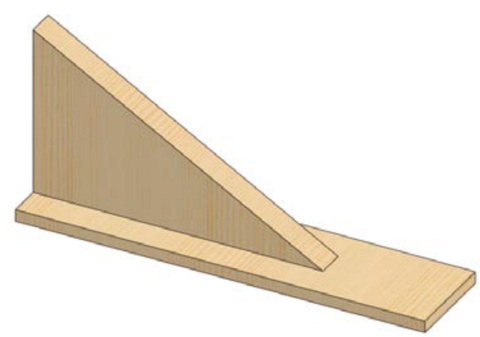
Ramp:
In the Front Plane, sketch and dimension the triangle shown here.



Extrude the triangle using the midplane option, with a thickness of 1.2 inches.

In the Top Plane, using the Corner Rectangle Tool, draw a rectangle. Add a midpoint relation between the left edge of the rectangle and the origin. Add the two dimensions shown, and extrude the rectangle down 0.5 inches.

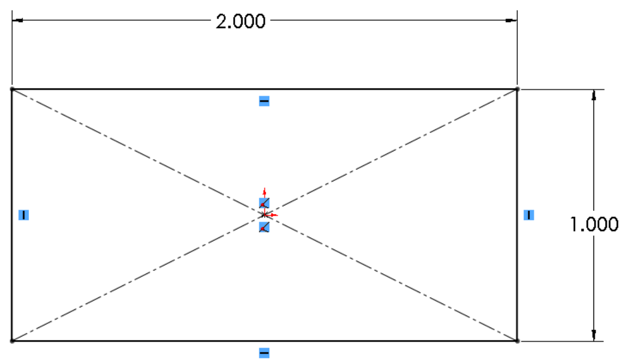


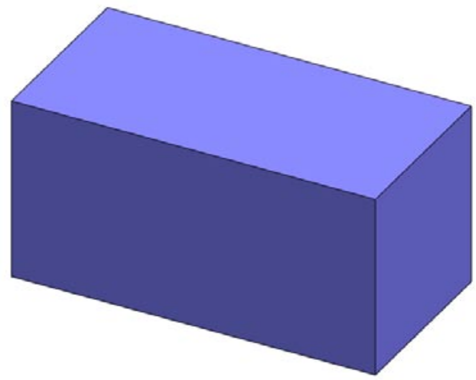


Modify the material/appearance as desired (shown here as Pine). Save this part with the name "Ramp".

Block:

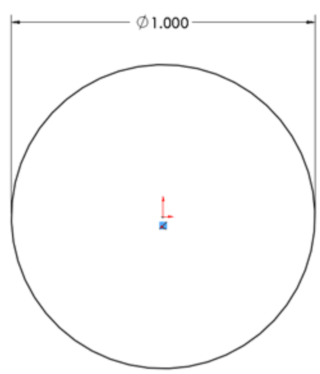
Sketch a 1-inch by 2-inch rectangle in the Front Plane, centered at the origin. Extrude the square with the midplane option, to a total thickness of one inch. Modify the appearance as desired (the material is not important), and save the part as "Block".

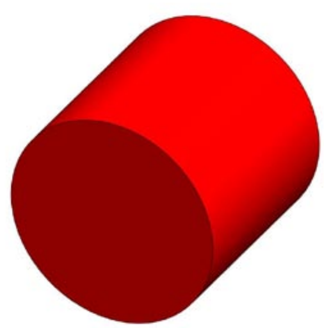




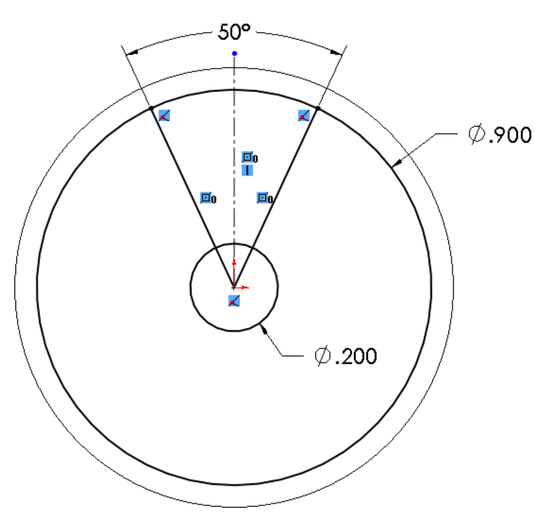
Roller 1:

Sketch and dimension a one-inch diameter circle in the Front Plane. Extrude the circle with the midplane option, to a total thickness of one inch. Set the material of the part as PVC Rigid. Modify the color of the part as desired (overriding the default color of the material selected).

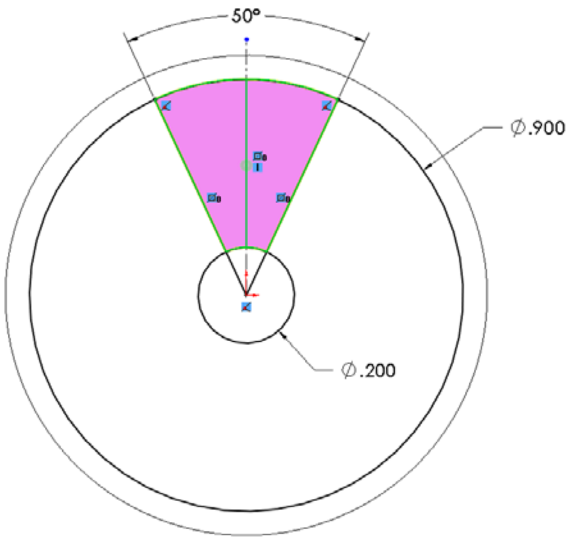


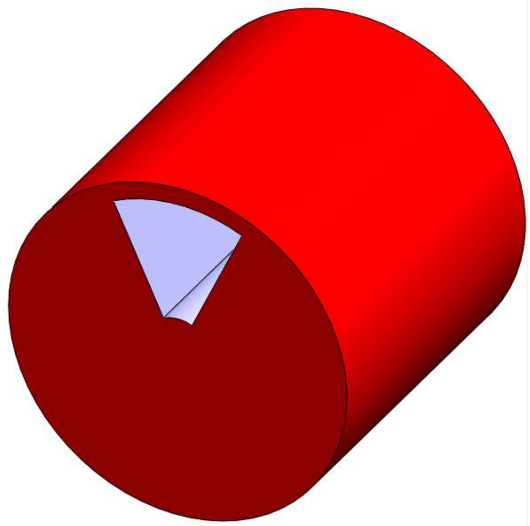


Open a new sketch on the front face of the cylinder. Add and dimension the circles and lines as shown here (the part is shown in wireframe mode for clarity). The two diagonal lines are symmetric about the vertical centerline.

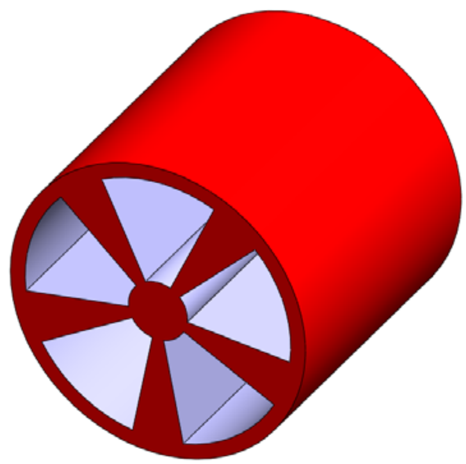


Extrude a cut with the Through All option, with the sketch contours shown selected. If desired, change the color of the cut feature.

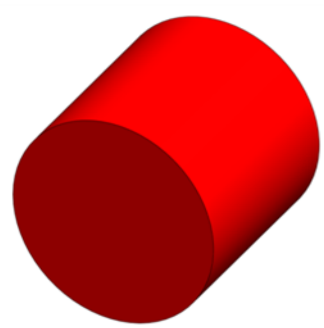




Create a circular pattern of the extruded cut features, with five equally-spaced cuts. Save the part with the name "Roller 1".



Roller 2:
Modify Roller 1 by suppressing the extruded cut (the circular pattern will be automatically suppressed, as well). Save the modified file as "Roller 2."

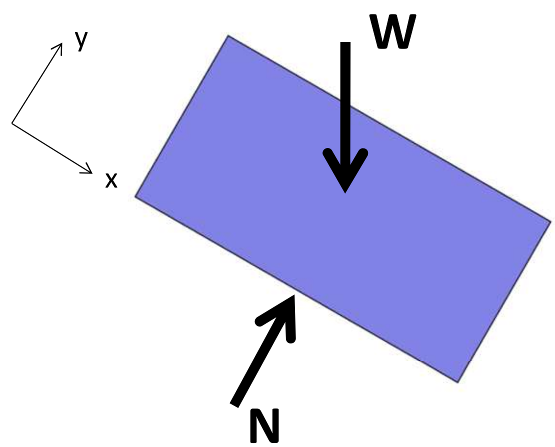


Roller 3:
Modify Roller 1 by changing the material to Plain Carbon Steel. Save the modified part as "Roller 3".

Roller 4:
Modify Roller 2 by changing the material to Plain Carbon Steel. Save the modified part as "Roller 4".

SIMULATION 1: What will be the motion of the block on the plane if there is no friction? Can we calculate the velocity of the block at the bottom of the ramp if we know its starting position?

While it is obvious that the block will slide down the ramp if there is no friction, this simulation is a good starting point for introducing Newton's Laws of Motion. A free body diagram (FBD) is often used in mechanics to show the forces acting on a body. A FBD of the block is shown here:



Two forces act on the body: the weight W and the normal force N which is caused by the contact between the block and the ramp. Newton's First Law states that unless a body is subjected to an unbalanced force, then the body remains at rest or traveling at a constant speed. Newton's Second Law states that a body subjected to an unbalanced force will accelerate in the direction of the unbalanced force, and the magnitude of the acceleration times the body's mass will equal the unbalanced force (F = ma). These laws can be applied to the components of the applied forces as well. If we set the coordinate system so that the x-direction is down the ramp, then we can sum the forces perpendicular to the ramp (the y-direction) as:

$$\Sigma F_y = N - W\cos\beta = 0 \qquad (1)$$

Where β is the ramp angle (30^o for this example). Here we know that the sum of the forces must equal zero, since the block cannot accelerate through the surface of the ramp. In the x-direction,

$$\Sigma F_x = W\sin\beta = m a_x \qquad (2)$$

Since the weight is equal the mass m times the gravitational acceleration g, the acceleration in the x-direction a_x will be:

$$a_x = g \sin \beta \quad (3)$$

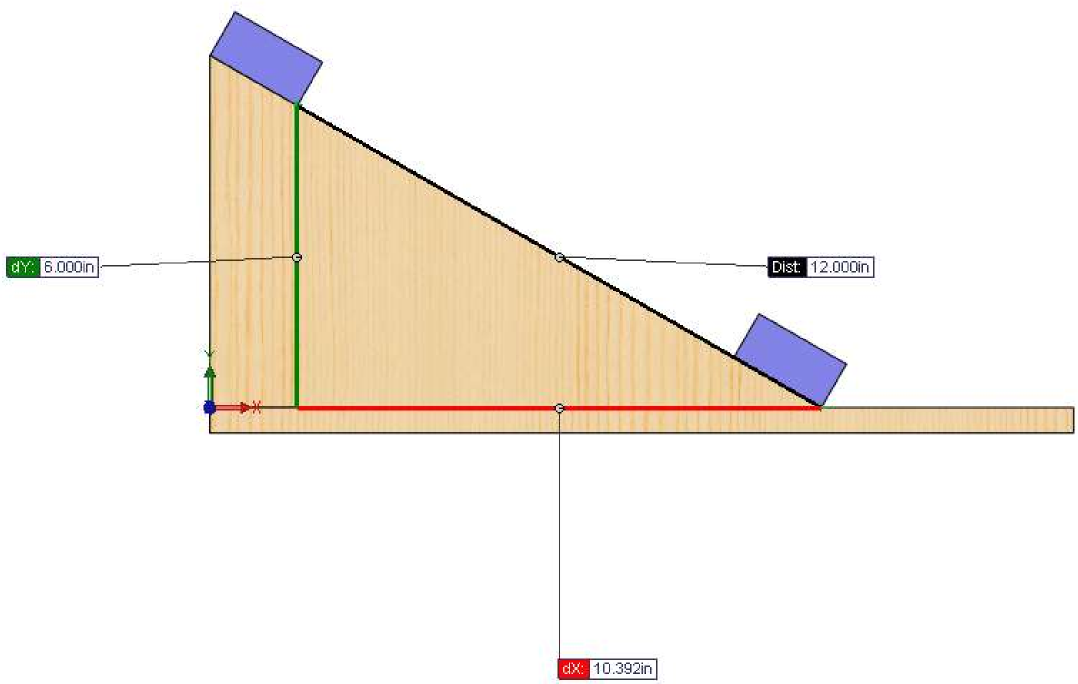
The acceleration is integrated with respect to time to find the velocity in the x-direction:

$$v_x = \int g \sin \beta \; dt = g \sin \beta \, t + v_{xo} \quad (4)$$

Where v_{xo} is the initial velocity in the x-direction. The velocity is integrated to find the distance travelled in the x-direction:

$$x = \int (g \sin \beta \, t + v_{xo}) \; dt = \frac{g}{2} \sin \beta \, t^2 + v_{xo} t + x_0 \quad (5)$$

Where x_0 is the initial position. If we measure x from the starting position, then x_0 is zero. If the block is initially at rest, then v_{xo} is also zero. In our simulation, we will place the front edge of the block six inches above the bottom of the ramp. Therefore, the block will slide a distance of 12 inches:



Knowing the distance travelled in the x-direction, and entering the numerical values of g as 386.1 in/s^2 and of $\sin \beta$ of 0.5 (sin of 30^o), we can solve Equation 5 for the time it takes the block to slide to the bottom:

$$12 \text{ in} = \frac{386.1 \text{ in/s}^2}{2} (0.5) t^2 \quad (6)$$

or

$$t = 0.353 \text{ s} \tag{7}$$

Substituting this value into Equation 4, we find the velocity at the bottom of the ramp:

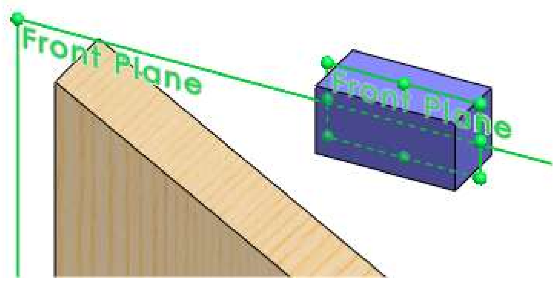
$$v_x = 386.1 \frac{\text{in}}{\text{s}^2}(0.5)(0.353 \text{ s}) = 68.1 \ \frac{\text{in}}{\text{s}} \tag{8}$$

We will now perform the simulation.

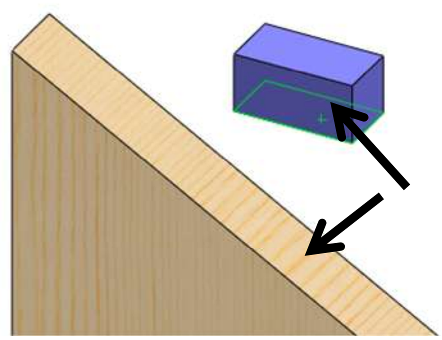
Open a new assembly. Set the units for the assembly to IPS (inch, pound, second) if SI units are your defaults. Insert the part "Ramp" into the assembly by clicking the check mark, which will align the ramp's coordinate system with that of the assembly's.

Insert the part "Block" into the assembly. Add three mates between the ramp and the roller:

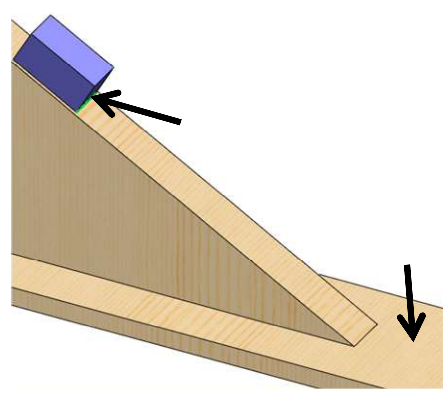
- ***A coincident mate between the Front Plane of the block and the Front Plane of the ramp,***



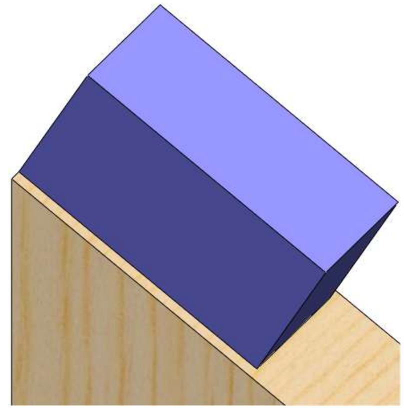
- ***A coincident mate between the bottom of the block and the inclined surface of the ramp, and***



- ***A distance mate between the leading edge of the block and the flat surface of the ramp. The distance should be 6.0 inches.***

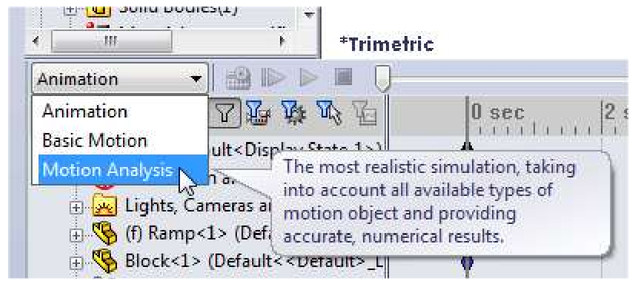


The block should now be positioned at the top of the ramp, as shown here.

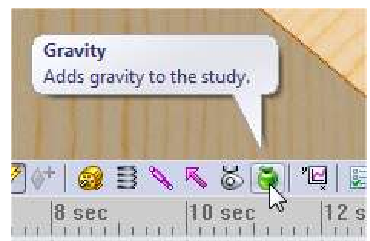


Click the Motion Study 1 tab. Activate the SolidWorks Motion Add-In from Tools: Add-ins from the main menu.

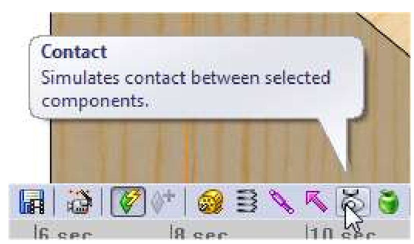
Change the type of simulation from Animation to Motion Analysis.

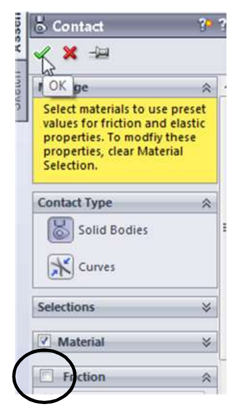


Click the Gravity icon. In the PropertyManager, click Y for the direction of gravity and click the check mark.

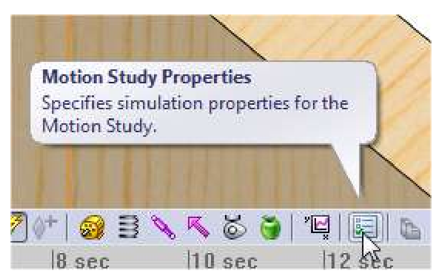


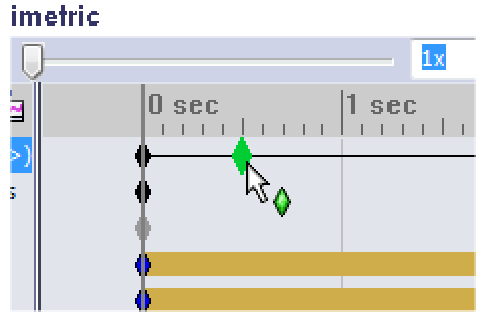
Click the Contact icon. Click on the block and the ramp to set contact between the two bodies. Clear the Friction box, and click the check mark.



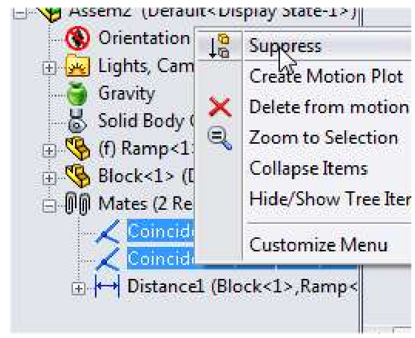


Click the Motion Study Properties icon. In the PropertyManager, set the Frame rate to 500 (frames per second), and check the box labeled "Use Precise Contact". Click the check mark.

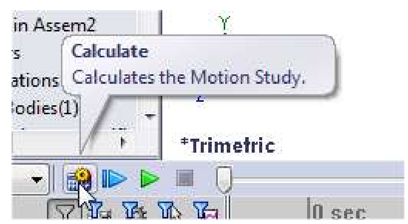




Since we have calculated that the block will reach the bottom of the ramp in less than ½ second, drag the diamond-shaped "key" at the top of the timeline from the default duration of 5 seconds to 0.5 seconds. (You may want to expand the timeline by clicking the Zoom In icon at the right edge of the timeline.)



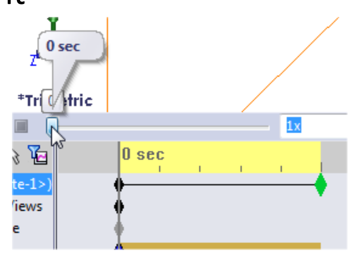
In the Motion Manager, select the three mates. Right-click, and select Suppress. (The mates could also be deleted. However, by suppressing them, the block can be returned to its starting position if necessary by unsuppressing the mates.)

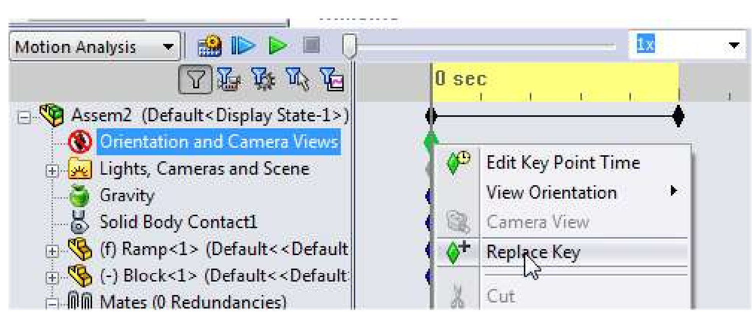


Click the Calculator icon to run the simulation.

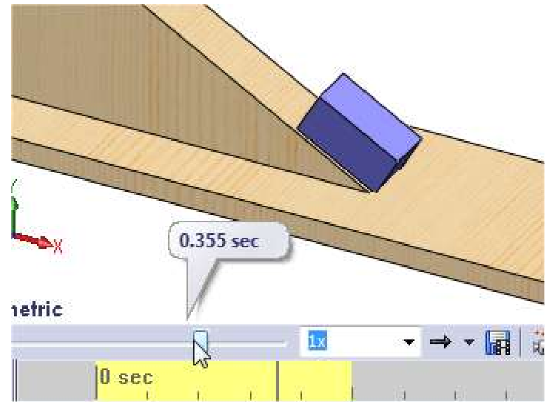
The block will now slide down the ramp and bounce off of the flat surface at the bottom.

After the simulation has been calculated, you can replay it without performing the calculations again by clicking the Play from Start icon. Note that the view orientation and zoom will always revert back to the model view that was active when the simulation was performed. If you would like to view the simulation from a different orientation or zoom level, there are two ways to do this. The first is to drag the time bar back to the beginning of the simulation, change to the desired viewing settings, and click the Play (not Play from Beginning) icon. The other is to change the view setting within the simulation. To do this, move the time bar back to the beginning of the simulation, switch to the desired view settings, and then right-click the key beside "Orientation and Camera Views" and select Replace Key.



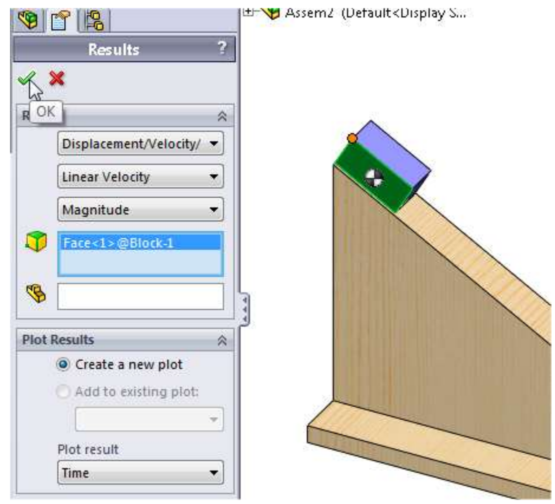


You can drag the time bar to determine the approximate time at which the block reaches the bottom. The time shown here, 0.355 seconds, is close to that calculated in Equation 7.

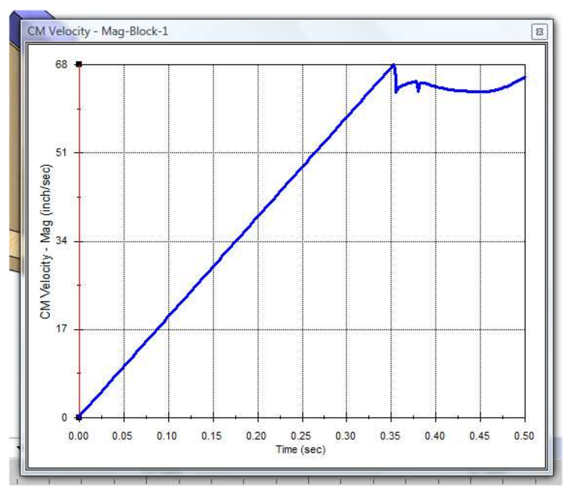


To get the velocity at the bottom of the ramp, we can create a graph.

Select the Results and Plots icon. In the PropertyManager, select Displacement/ Velocity/Acceleration: Linear Velocity: Magnitude. Click on a face of the block. Click the check mark.

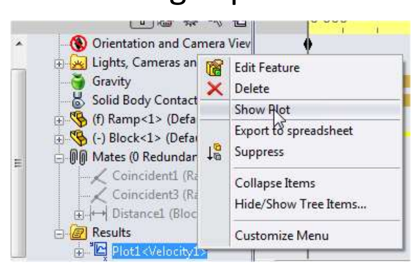


A graph of the velocity of the center of mass of the block vs. time is created. Note that the velocity peaks at 68 in/s and decreases when the block impacts the flat surface at the bottom of the ramp.



The graph can be edited by right-clicking a part of the graph and selecting Properties. Here the chart area is set to plain white and the curve is set to a heavier line type. The axes can also be edited to show more decimal places, but that is not necessary for our analysis. If more exact values are needed, the data values can be exported to a comma-separated-values file (.CSV) by right-clicking the chart and selecting Export CSV. The CSV file can be opened in Excel.

If you want to hide the graph, you can do so by right-clicking and selecting Hide. To show the graph again, right-click its name under Results in the MotionManager tree and choose Show Plot.



Before leaving this simulation, let's look at another method for calculating the final velocity. The principle of conservation of energy tells us that the energy of the block at the top of the ramp will equal the energy of the block at the bottom of the ramp. The total energy consists of two components – potential and kinetic energy. The potential energy is equal to the weight (mass times g) of the block times the distance above a reference plane. The kinetic energy is equal to ½ the mass of the block times the magnitude of its velocity squared. If we call the top of the ramp position A and the bottom of the ramp position B, then

$$\mathrm{mg}h_{\mathrm{A}} + \frac{1}{2}\mathrm{m}v_{\mathrm{A}}^2 = \mathrm{mg}h_{\mathrm{B}} + \frac{1}{2}\mathrm{m}v_{\mathrm{B}}^2 \tag{9}$$

Since the block is originally at rest, the velocity at position A is zero. Also, we can factor out the mass:

$$\frac{1}{2}v_{\mathrm{B}}^2 = \mathrm{g}(h_{\mathrm{A}} - h_{\mathrm{B}}) \tag{10}$$

So

$$v_{\mathrm{B}} = \sqrt{2\mathrm{g}(h_{\mathrm{A}} - h_{\mathrm{B}})} \tag{11}$$

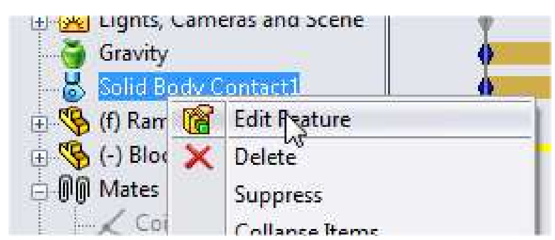
Note that the choice of the reference plane is unimportant; only the height difference between positions 1 and 2 is significant. In this case the height difference is 6 inches. Substituting this value and the numerical value of g into Equation 11, we obtain

$$v_{\mathrm{B}} = \sqrt{2\left(386.1\,\frac{\mathrm{in}}{\mathrm{s}^2}\right)(6\ \mathrm{in})} \; = \; 68.1\,\frac{in}{s} \quad \checkmark \tag{12}$$

SIMULATION 2: What will be the motion of the block on the plane if friction is included? Can we calculate the velocity of the block at the bottom of the ramp?

For this simulation, we will simply try various values of the friction coefficient μ and then try to interpret the results.

In the MotionManager tree, right-click the Contact specification and choose Edit Feature.

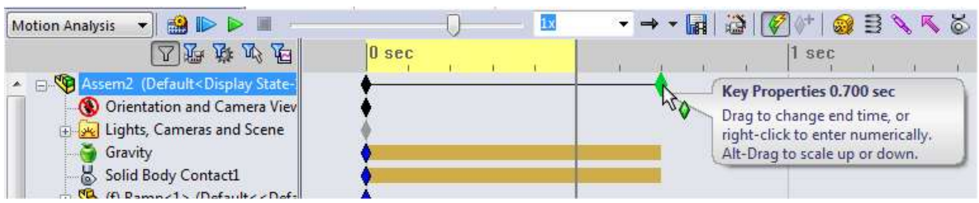


In the PropertyManager, clear the Material checkbox and check the Friction box. Set the coefficient of kinetic friction to 0.10. Click the check mark.

For simplicity, we will assume that the static and kinetic coefficients of friction are the same.

Run the simulation. Estimate the maximum speed and the time required to reach the bottom of the ramp from the graph.

Repeat the analysis, increasing the coefficient of friction by 0.10 each time. If the block does not reach the bottom of the ramp before the simulation ends, add more time by dragging the top key of the timeline to the right and re-calculating.



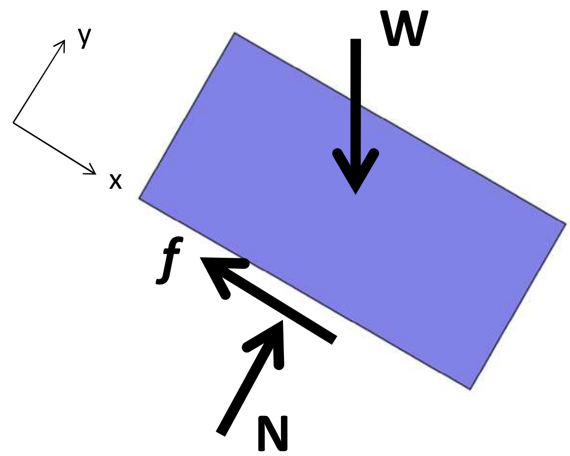
Here are typical results:

Coefficient of Friction	Velocity at Bottom of Ramp, in/s	Time to Reach Bottom of Ramp, s
0	68	0.36
0.10	62	0.39
0.20	55	0.44
0.30	47	0.51
0.40	38	0.63
0.50	25	0.94
0.60	*	*

*Does not reach bottom after 5 seconds. Block moves slightly, but maximum velocity is less than 0.5 in/s.

As expected, the higher the friction, the more time that is required to slide down the ramp, and the lower the speed at the bottom. When the friction reaches a value of 0.60, the block appears to be stationary, although slight motion is observed (more about this later). There seems to a threshold between values of 0.5 and 0.6 above which the block will not slide. We will now attempt to find that value analytically.

Let's look at the FBD, which has been modified to include the friction force f:



Note that the friction force opposes the direction of motion (or impending motion). The value of the friction force is the value needed to keep the block in equilibrium, up to a maximum value of μN, where μ is the coefficient of friction. Actually, the maximum friction force is $\mu_s N$, where μ_s is the static coefficient of friction. When the block is sliding, the friction force equals $\mu_k N$, where μ_k is the kinetic coefficient of friction. The static coefficient is usually slightly higher than the kinetic coefficient of friction, as it requires more force to start the block to move than to keep it moving. However, as noted earlier, we will use a single value of μ for simplicity.

If we sum the forces in the y-direction, then we get the same result as before (without friction):

$$\Sigma F_y = N - W\cos\beta = 0 \tag{13}$$

so

$$N = W\cos\beta \tag{14}$$

In the x-direction, the friction force is included in the equation:

$$\Sigma F_x = W\sin\beta - f = 0 \tag{15}$$

When no sliding occurs,

$$f = W\sin\beta \tag{16}$$

If the block is on the *verge* of sliding, then the friction force is at its maximum value:

$$f = \mu N = \mu W\cos\beta \tag{17}$$

Substituting this expression into Equation 16,

$$\mu W \cos\beta = W \sin\beta \tag{18}$$

so

$$\mu = \frac{\sin\beta}{\cos\beta} = \tan\beta \tag{19}$$

In other words, the value of μ must be greater than or equal to the tangent of the ramp angle in order to prevent motion. For our example, with a ramp angle of 30^{o}, a coefficient of friction of 0.577 or greater is required to prevent sliding. This result agrees with that of our simulation trials, which showed sliding at μ = 0.50 and no sliding at μ = 0.60.

When a friction coefficient value of 0.60 was used in our simulation, some slight movement of the block was seen. Also, when the blocks that did slide contacted the flat surface at the bottom of the ramp, the block appeared to penetrate the surface. Both of these observations are related to the way in which contacts are simulated in SolidWorks Motion. In addition to a friction value, stiffness parameters can be input in the contact's definition. The stiffness is based on the amount of deflection that is observed when the two bodies come into contact. In other words, the contact is simulated as a spring force. At each time interval, the algorithm checks to see if the two bodies penetrate one another. If so, the bodies are pushed apart with a force that varies with the depth of penetration. The amount of penetration can be limited by increasing the stiffness value or by decreasing the time intervals. Usually, the default settings will produce reasonable results, although some unexpected behaviors, such as the block gradually being "nudged" down the ramp by the contact forces, can be present in simulations involving contacts.

Returning to our equations of motion, when sliding occurs, Equation 15 becomes

$$\Sigma F_x = W \sin\beta - f = m a_x \tag{20}$$

The maximum friction force is given in Equation 17. Substituting this expression into Equation 20 and solving for the acceleration a_x,

$$m a_x = W \sin\beta - f$$

$$m a_x = W \sin\beta - \mu W \cos\beta$$

$$a_x = g(\sin\beta - \mu\cos\beta) \tag{21}$$

Integrating to get the velocity and position (and dropping the initial velocity and position terms, since they are zero),

$$v_x = g(\sin\beta - \mu\cos\beta)t \tag{22}$$

and

$$x = \frac{g}{2}(\sin\beta - \mu\cos\beta)t^2 \tag{23}$$

As an example, consider a friction coefficient of 0.30. When the block has slid 12 inches, Equation 23 can be solved for t:

$$12\text{in} = \frac{386.1\ \text{in/s}^2}{2}(\sin 30° - 0.3\cos 30°)t^2$$

$$t = \sqrt{\frac{2(12\text{in})}{386.1\ \text{in/s}^2(\sin 30° - 0.3\cos 30°)}} \ = \ 0.509\ \text{s} \tag{24}$$

Substituting this time value into Equation 22, the velocity at the bottom of the ramp is

$$v_x = 386.1\frac{\text{in}}{\text{s}^2}(\sin 30° - 0.3\cos 30°)(0.509\ s) = 47\ \frac{\text{in}}{\text{s}} \tag{25}$$

The energy method can also be used. Friction is a non-conservative force, which means that the work done by the friction is energy that is lost to the block. Therefore, the energy lost due to friction (the force integrated over its path) must be included in the energy equation (Equation 10):

$$\text{mg}h_A + \frac{1}{2}\text{m}v_A^2 - \int f\,dx \ = \text{mg}h_B + \frac{1}{2}\text{m}v_B^2 \tag{26}$$

The integral is evaluated as:

$$\int f\,dx = \int_0^L \mu W\cos\theta\,dx = L\,\mu\,\text{mg}\cos\beta \tag{27}$$

where L is the length of the path (12 inches) that the friction acts over. Equation 26 can be rearranged as:

$$g(h_A - h_B) - L\mu\,g\cos\beta \ = \frac{1}{2}v_B^2 \tag{28}$$

Solving for v_B:

$$v_B = \sqrt{g(h_A - h_B - L\mu\ \cos\beta)} \qquad (29)$$

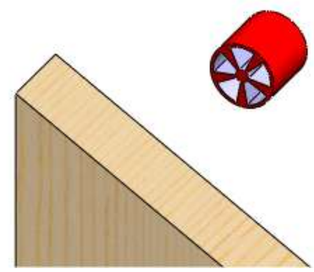
Entering the numerical values,

$$v_B = \sqrt{386.1\frac{\text{in}}{\text{s}^2}(6\text{ in} - 12\text{in}(0.3)(\cos 30°)} = 47\frac{in}{s} \qquad (30)$$

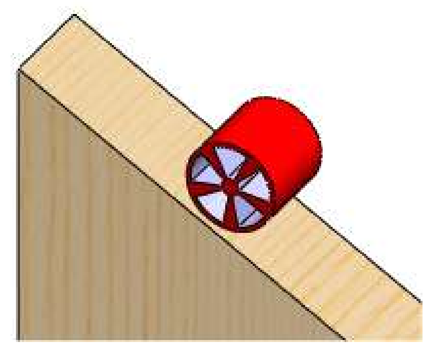
<u>SIMULATION 3:</u> How will the motion of a roller be different than that of the block?

We will begin by simulating a roller with no friction, with the assumption that its behavior will be similar to that of the sliding block.

Open a new assembly. Insert the ramp first, and place it at the origin of the assembly. Insert the Roller 1 part.



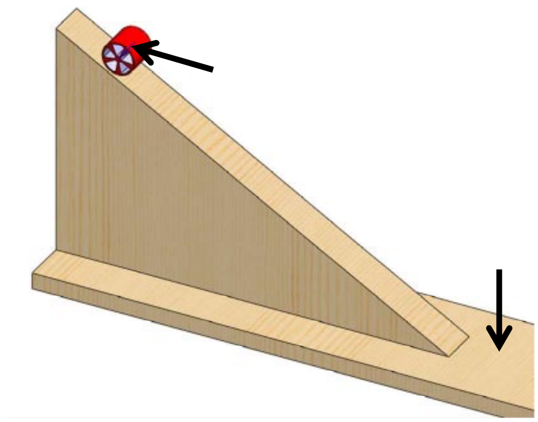
Add two mates between the ramp and the roller. Mate the Right Planes of both parts, and add a tangent mate between the cylindrical surface of the roller and the surface of the ramp.



The best way to set the correct height of the roller on the ramp is to add a mate defining the position of the axis of the roller.

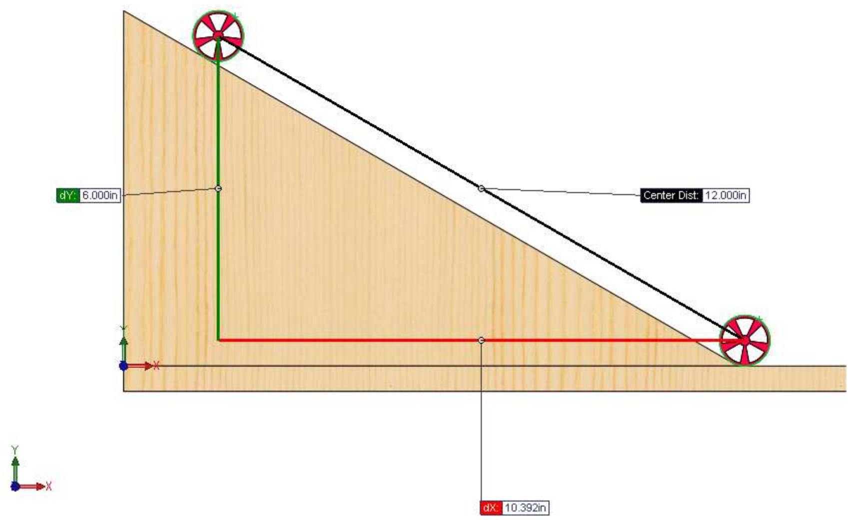
From the Heads-Up View Toolbar, select View: Temporary Axes.

This command turns on the display of axes that are associated with cylindrical features.



Add a distance mate between the roller's axis and the flat surface at the bottom of the ramp. Set the distance as 6.5 inches.

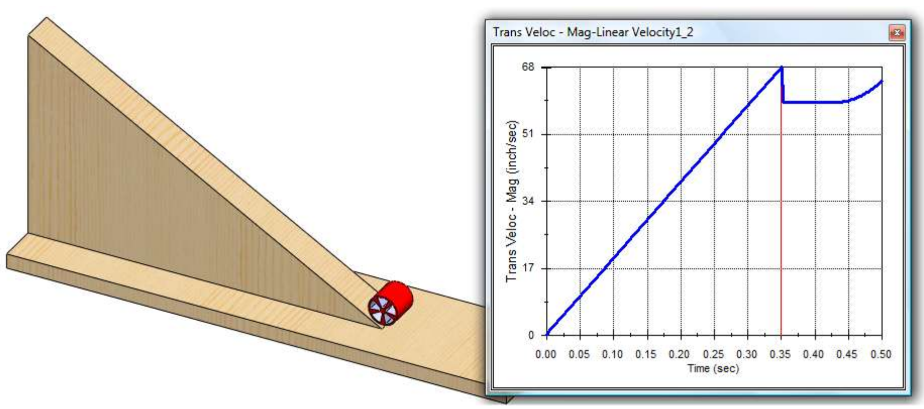
Since the radius of the roller is 0.5 inches, the axis will be 0.5 inches above the flat surface when the surface of the roller contacts that surface. Therefore, the vertical distance traveled by the roller will be 6.0 inches, which is the same as for the simulations with the blocks. The total distance traveled down the ramp will again be 12 inches.



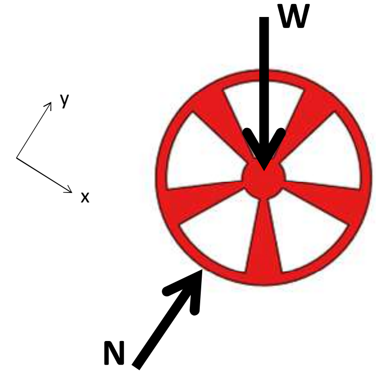
Turn off the temporary axis display. Switch to the Motion Study. Select SolidWorks Simulation as the type of analysis. Add gravity in the -y-direction, and add a solid body contact between the roller and the ramp, with the friction turned off. Set the frame rate to 500 and specify precise contact. Suppress the three mates, and run the simulation for 0.5 seconds.

Create a plot of the magnitude of the linear velocity of the roller vs. time.

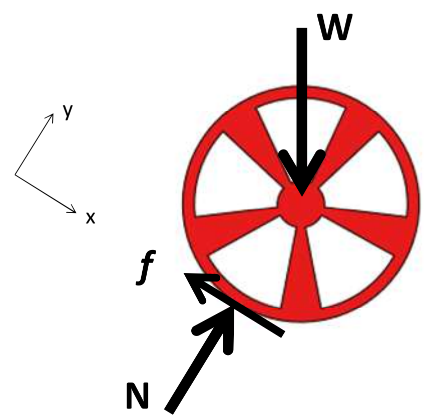
The resulting plot should be similar to that of the sliding block with no friction. The roller reached the bottom of the ramp in about 0.35 seconds, and the velocity at the bottom of the ramp is about 68 in/s.



The FBD of the roller is similar to that of the block on page 4. The only forces acting on the roller are the weight and the normal force, both of which act through the center of mass of the roller. Therefore, the analysis of the motion of the roller is exactly the same as that of the motion of the block, as long as there is no friction.



Now let's add friction, and examine the FBD:



Without friction, the motion of the roller is that of a particle. In mechanics, particles are defined as bodies for which all of the forces acting on the body act through the center of mass of the body, and the equations of motion associated with summing forces in the x- and y-directions completely define the body's motion. However, when one or more of the forces acting on a body does not act through the body's center of mass, such as the friction force, then the body is considered to be a rigid body rather than a particle[1], and there is an equation of motion related to the sum of the moments acting on the body:

$$\Sigma \mathrm{M_c} = \mathrm{I_c}\alpha \tag{31}$$

where $\mathrm{I_c}$ is the mass moment of inertia of the roller about an axis through the center of mass, and α is the angular acceleration of the roller. The mass moment of interia of a body is defined as:

$$\mathrm{I_c} = \int r^2\, dm \tag{32}$$

[1] It should be noted here that the block with friction present is also a rigid body, as the friction force does not act through the center of mass of the block (FBD on page 12). However, the normal force N is actually the resultant of a pressure distribution between the block and ramp and this pressure can re-distribute to prevent the block from rotating. The roller contacts the ramp at a single location (actually a small area that is idealized as a single edge) and so the location of the normal force cannot change. Of course, if the block is tall and/or the ramp is steep, then the block can rotate. Just before rotating, the normal force is concentrated at the front edge of the block. At the point, the motion of the block is very similar to that of the roller.

That is, the mass moment of inertia is a measure of how the mass in a body is distributed relative to the axis of rotation. If two rollers have the same total mass, the one with more of its mass toward the outer edge of the roller will have a higher mass moment of inertia. Values of I_c are tabulated for simple shapes, such as a solid cylinder, and SolidWorks can be used to find I_c for more complex shapes.

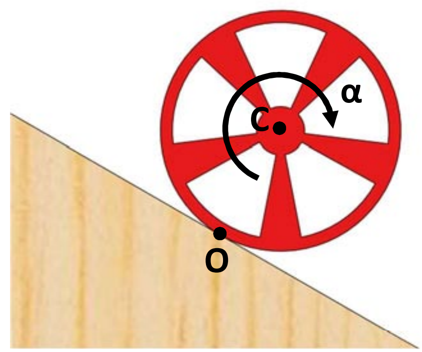
Let's consider the case of no sliding. In this case, the three equations of motion are:

$$\Sigma F_y = N - W\cos\beta = 0 \tag{33}$$

$$\Sigma F_x = W\sin\beta - f = m a_x \tag{34}$$

$$\Sigma M_c = f\,r = I_o \alpha \tag{35}$$

If there is no slipping, then the relative velocity of the roller relative to the ramp is zero at the point where the two bodies are in contact (point O). Since the ramp is stationary, this leads to the observation that the velocity of point O is also zero.



Since point O is the center of rotation of the roller, the tangential acceleration of the center of the roller (a_x) can be written as:

$$a_x = r\alpha \tag{36}$$

Substituting this expression into Equation 35 and solving for the friction force,

$$f = W\sin\beta - mr\alpha \tag{37}$$

Substituting this expression into Equation 35 and solving for α,

$$(W\sin\beta - mr\alpha)\,r = I_o\alpha$$

$$W(\sin\beta)r = I_o\alpha + mr^2\alpha$$

$$\alpha = \frac{W(\sin\beta)r}{I_o + mr^2} \tag{38}$$

The numerical values for this simulation are shown in the spreadsheet below. Note that the mass and mass moment of inertia calculated in SolidWorks are in units of pounds and pounds-in^2. In order to use a consistent set of units, both of these values are divided by g (386.1 in/s^2). The value of angular acceleration is calculated as 243.4 rad/s^2.

	A	B	C	D	E	F	G
1	W from SW	0.017163	lb		Ang. Acceleration	243.4	rad/s^2
2	I from SW	0.0025151	lb-in^2				
3	Ramp Angle	30	deg		Friction force	3.17E-03	lb
4	r of Roller	0.5	in				
5					Coefficient of Friction		
6	Convert to consistent units:				Required for No Slip	0.21	
7	m	4.45E-05	lb-s^2/in				
8	I	6.51E-06	lb-in-s^2		Length of Ramp	12	in
9					Revolutions to Bottom	3.820	
10	Normal force	1.49E-02	lb		Radians to Bottom	24	rad
11					Time to Bottom	0.444	s
12					Final Angular velocity	108.1	rad/s
13					Final Linear velocity	54.0	in/s

To find the linear velocity, the angular acceleration is integrated twice to find the angular velocity and angular position change:

$$\omega = \int \alpha dt = \alpha t + \omega_o \tag{39}$$

$$\theta = \int \omega dt = \frac{\alpha}{2} t^2 + \omega_o t + \theta_o \tag{40}$$

Since the initial angular velocity and position are zero, those terms can be ignored. To roll from the top to the bottom of the ramp, the roller moves linearly 12 inches. With no slip, this distance corresponds to 3.82 revolutions (24 radians). This allows us to solve for the time to reach the bottom of the ramp and the angular velocity at the bottom of the ramp:

$$24\ \text{rad} = \frac{243.4\ \text{rad/s}^2}{2} t^2$$

$$t = \sqrt{\frac{2(24\ \text{rad})}{243.4\ \text{rad/s}^2}} = 0.444\ \text{s} \tag{41}$$

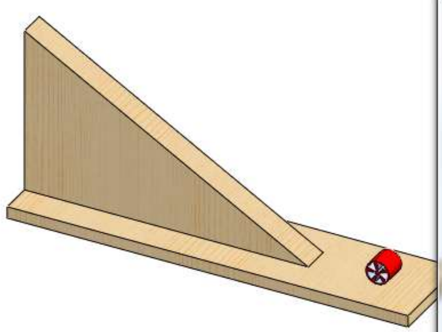
$$\omega = \left(243.4 \frac{\text{rad}}{\text{s}}\right)(0.444\ \text{s}) = 108.1\ \frac{\text{rad}}{\text{s}} \tag{42}$$

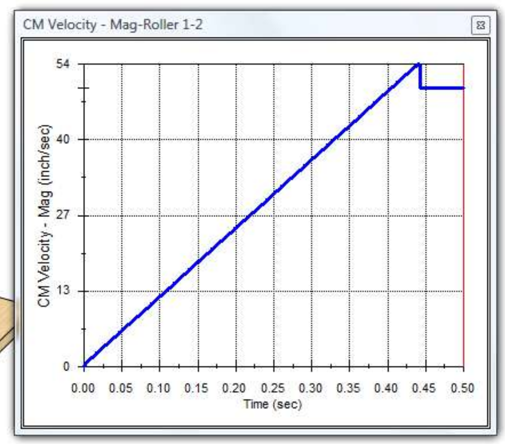
The linear velocity of the roller is simply the angular velocity times the radius, so

$$v = \left(108.1\ \frac{\text{rad}}{\text{s}}\right)\left(0.5\frac{\text{in}}{\text{rad}}\right) = 54.0\ \frac{\text{in}}{\text{s}} \tag{43}$$

Let's run the simulation to verify our result. Note that in our spreadsheet, we have calculated the friction force from Equation 37 and the normal force from Equation 33. Since the maximum friction force is equal to the coefficient of friction times the normal force, we can find the minimum coefficient of friction required to prevent slipping by dividing the friction force by the normal force. For our system, this value is 0.21. (It is interesting to note that this value is much lower than the value of the friction coefficient required to prevent sliding is no rotation is present, 0.58.)

Right-click on the contact in the MotionManager tree, and select Edit Feature. Set the coefficient of friction to 0.25. Calculate the simulation.

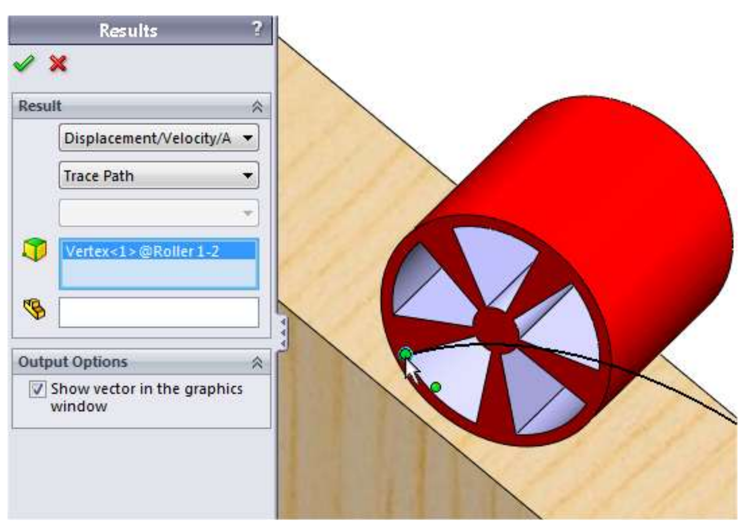




The resulting velocity plot confirms our calculated velocity at the bottom of the ramp, 54 in/s.

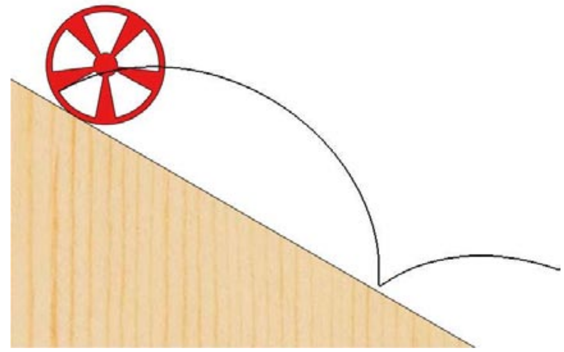
This result can also be verified with an energy analysis. There is an additional kinetic energy term, $1/2(\mathrm{I}_0)\omega^2$, that must be included due to the rotation of the roller.

To confirm that the roller is not slipping, we can trace the position of a single point on the roller.



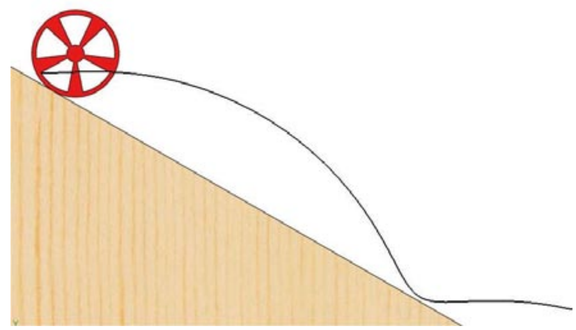
Select the Results and Plots Tool. Define the plot as Displacement/ Velocity/Acceleration: Trace Path. Click on a point near the outer rim of the roller (not on a face, but on a single point). Click the check mark.

The trace path shows a sharp cusps where the point's velocity approaches zero (it will not become exactly zero unless the point is on the outer surface of the roller). For comparison, repeat the analysis with a lower friction coefficient.



Change the friction coefficient to 0.15 and recalculate the simulation.

This time, the trace paths shows smooth curves when the point is near the ramp's surface, indicating that sliding and rolling are taking place simultaneously.



<u>SIMULATION 4:</u> How do the mass properties of the roller affect its behavior?

Delete the trace path from the Results in the MotionManager tree. Switch from the Motion Study to the Model by clicking the Model tab near the bottom of the screen. Click on the roller to select it. From the main menu, select File: Replace. Browse to find Roller 3, and click the check mark. When asked if you want to replace the affected mates, click the check mark.

Recall that Roller 3 has the same geometry as Roller 1, but is much heavier (made of steel instead of PVC).

Change the friction coefficient to 0.25, and run the simulation.

The velocity at the bottom of the ramp is unchanged (54 in/s).

Now replace Roller 3 with Roller 2 and repeat the analysis.

This time, the final velocity is slightly greater: 55 in/s. Is this a significant difference or just a numerical round-off difference?

To answer this, find the mass properties of Roller 3 from SolidWorks. Insert these values into the calculations described earlier and you will see that the calculated final velocity is indeed greater:

	A	B	C	D	E	F	G
1	W from SW	0.0368866	lb		Ang. Acceleration	257.4	rad/s^2
2	I from SW	0.0046108	lb-in^2				
3	Ramp Angle	30	deg		Friction force	6.15E-03	lb
4	r of Roller	0.5	in				
5					Coefficient of Friction		
6	Convert to consistent units:				Required for No Slip	0.19	
7	m	9.55E-05	lb-s^2/in				
8	I	1.19E-05	lb-in-s^2		Length of Ramp	12	in
9					Revolutions to Bottom	3.820	
10	Normal force	3.19E-02	lb		Radians to Bottom	24	rad
11					Time to Bottom	0.432	s
12					Final Angular velocity	111.2	rad/s
13					Final Linear velocity	55.6	in/s

Examining Equation 38, we see that the angular acceleration depends on the quantity W/I_o. Comparing the four rollers, we see that this quantity is a function of geometry, and not of total mass:

Roller	Weight, lb	I_o, lb-in^2	W/I_o, 1/in^2
1 (PVC, slotted)	0.017163	0.002515	6.82
2 (PVC, solid)	0.036887	0.004611	8.00
3 (Steel, slotted)	0.102975	0.015091	6.82
4 (Steel, solid)	0.221320	0.027665	8.00

To verify these results, you can stage a race between the four rollers. The two solid rollers will get to the bottom at the same time, just ahead of the two slotted rollers.